**Introduction: Even and Odd Functions**

Even and odd functions are two types of functions that are covered in any pre-calculus courses. They are important functions that have a number of basic properties. First, let’s define what an even and odd function is.

**Def 1**: A function is an even function of x if

**Def 2**: A function is an odd function of x if

Geometrically, an even function is symmetric about the y-axis. In other words, the graph is unchanged if reflected about the y-axis. An odd function is symmetric about the origin meaning that the graph is unchanged if it is rotated 180° about the origin. Some examples of even functions include. Some examples of odd functions include . It is also important to note that not all functions can be classified as either even or odd. It is only those that satisfy the relation in definition 1 or definition 2.

Even and odd functions also satisfy a number of basic properties.

1. The only function that is both even and odd is the constant function 0.
2. The sum of an even and odd function is neither even nor odd unless one of the functions is equal to 0.
3. The sum of two even functions is an even function.
4. The sum of two odd functions is an odd function.
5. Any constant multiple of an even/odd function is even/odd function, respectively.
6. The product of two even functions is an even function.
7. The product of two odd functions is an even function.
8. The product of an even function and an odd function is an odd function.

**The Blip of the Blop and Our Seminar Work**

In our mathematics seminar, we are exploring the interchanging of the order of mathematical operations. There are cases in which this is always true, sometimes true, or never true. For example, we began the semester working with basic mathematical operations such as addition, multiplication, and exponentiation. We wanted to discover when, if at all, some of these operations could be interchanged. Below are a few examples of the types of problems we considered.

We asked what values of x and y satisfy each of the following equations



For (1), in words, we are looking at when the sum of the square is equal to the square of the sum. For (2), we are looking at when the sum of the cube is equal to the cube of the sum.

For (3) we are looking at when the exponential of the product is equal to the product of the exponentials. Notice how the orders of these operations are being switched. We are looking at when the **blip** of the **blop** is equal to the **blop** of the **blip**.

**Reciprocal Even and Reciprocal Odd**

As we progressed along in our seminar, we became interested in looking at a particular type of relation which we have named and defined all on our own.

**Def 3:** A function is even with respect to reciprocal or reciprocal even if

**Def 4:** A function is odd with respect to reciprocal or reciprocal odd if

This project is an extension of our seminar work in which we are looking at when functions satisfy the relationship in definition 3 and definition 4.

It is important to notice the parallel between these functions and our discussion of even/odd functions. Functions usually called even would be called even with respect to negation or negative even in this terminology and, similarly, odd functions would be called odd with respect to negation or negative odd. A function that is reciprocal even has the same value at a number x as it does for the reciprocal of x, namely . A function that is reciprocal odd satisfies the property that the function of the reciprocal equals the reciprocal of the function. (The Blip of the Blop!!!)

These types of functions display similar properties to even/odd functions, but with some important changes.

1. The only function that is both reciprocal even and reciprocal odd is the constant function 1 or -1.
2. The sum of a reciprocal even and reciprocal odd function is reciprocal even if and only if the reciprocal odd function is which is also reciprocal even.
3. **The sum of a reciprocal even and a reciprocal odd function is reciprocal odd**
4. The sum of two reciprocal even functions is reciprocal even.
5. The sum of two reciprocal odd functions is never reciprocal odd.
6. Any constant multiple of a reciprocal even function is reciprocal even, but the only case in which a multiple of a reciprocal odd function is reciprocal odd is if the multiple is 1.
7. The product of two reciprocal even functions is reciprocal even.
8. The product of two reciprocal odd functions is reciprocal odd.
9. The product of a reciprocal even function and a reciprocal odd function is reciprocal even/odd if and only if the reciprocal odd/even function is which is also reciprocal even.

The proofs to these statements are straightforward and are not included in our poster, but are available for anyone who would like to see them.